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ON LIMITING DISTRIBUTION FOR TIME FAILURE OF TWO DISSIMILAR UNIT STANDBY REDUNDANT SYSTEM WITH REPAIR AND PREVENTIVE MAINTENANCE

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On Limiting Distribution for Time Failure of Two Dissimilar Unit Standby Redundant System with Repair and Preventive Maintenance

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Key Words - Reliability

Purpose : Report of a derivation

Special Math needed : Probability, Laplace transform

Results useful to : Theoreticians and theoretically inclined reliability engineers

Abstract :

This paper discusses the case of a two-dissimilar unit redundant repairable system under moving policy.We shall prove that the system reliability function will tend to exponential one under the condition that the mean life time of the system is much larger than the mean repair time and the maintenance time.

INTRODUCTION

In practice there are three maintenance policies defined as :

i) - Moving maintenance; the preventive maintenance performs only if the reserve element is in standby . If the reserve element is under repair or preventive maintenance, then the preventive maintenance performs only after completion repair or preventive maintenance of the reserve element.

ii) - Economic maintenance; when the inspection moment is attained, the preventive maintenance performs only if there exist a reserve element in standby. If there is no element in standby, the preventive mainte-nance does not perform and the operative elements are operating usually up to the failure moment of one of them .

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iii) - Difficult maintenance; in which the preventive maintenance performs independently on the state of the reserve element which may be in standby, repair or preventive maintenance

In the case of standby redundant system with repair mentioned in [1] and [2], it is proved that the repairable distribution function will tend to exponential one when the mean repair time is very composed to the mean life time of the system. The limiting failure time distribution of standby redundant repairable system of two units and imperfect switchover is studied by [3], [4]. The limiting distribution function of time failure of two similar units with preventive maintenance for policy (i),(ii) and (iii) is studied by [5], [6]. In this paper we study the limiting distribution function for time failure of two dissimilar units standby redundant system with preventive maintenance under Moving maintenance policy .

ASSUMPTIONS AND DEFINITIONS

1. The failure time distribution functions for the two units are $F_i(x)$ $(i = 1,2)$.
2. The replacement interval to inspection moment is a random variable with distribution function $U_i(x)$ $(i = 1,2)$.
3. The repair and maintenance times are random variables $G_i(x)$, $V_i(x)$ and we assume that $V_i(x) > G_i(x)$ $(i = 1,2)$.
4. The time required for a unit to replace another one connected to repair or maintenance is negligible.
5. After repair and preventive maintenance each unit recovers its function perfectly and takes its place in standby immediately, if not required to operate .

6. All random variables are mutually independent and non-negative.

7. Let $X(t)$ be the state variable of the system at instant t,

$X(t) = 0$: The 1 st unit operates and the 2 nd unit in standby .

$X(t) = 1$: The 1 st unit under repair and the 2 nd unit operates .

$X(t) = 2$: The 1 st unit is under preventive maintenance and the 2 nd unit operates .

$X(t) = 3$: The 2 nd unit is under repair and the 1 st unit operates.

$X(t) = 4$: The 2 nd unit is under preventive maintenance and the 1 st unit operates .

8. R_j - denote the probability that the system is in state j , $(j = 1,2,3,4)$ at the instant $t = 0$ will work smoothly to the instant t .

9. $R(t)$ - denote the probability that the system in state 0 at instant $t = 0$ will work smoothly to the instant t.

FORMULATION OF EQUATIONS

Assuming that the points $X_i(t)$, $(i = \overline{0,4})$ are regeneration points, it is easy to prove that $R(t)$, $R_j(t)$, $(j = \overline{1,4})$ satisfy the following probabilistic integral equations .

$$R(t) = \overline{F}_1(t)\,\overline{U}_1(t) + \int_0^t \overline{U}_1(x)\, R_1(t-x)\, dF_1(x) + \int_0^t \overline{F}_1(x)\, R_2(t-x)\, dU_1(x)$$

$$R_1(t) = \overline{F}_2(t)\,\overline{U}_2(t) + \int_0^t R_3(t-x)\, G_1(x)\, \overline{U}_2(x)\, dF_2(x) +$$

$$+ \int_0^t R_4(t-x)\, G_1(x)\, \overline{F}_2(x)\, dU_2(x) + \int_0^t R_4(t-x)\, \overline{F}_2(x)\, U_2(x)\, dG_1(x)$$

$$R_2(t) = \overline{F}_2(t)\,\overline{U}_2(t) + \int_0^t R_3(t-x)\, V_1(x)\, \overline{U}_2(x)\, dF_2(x) +$$

$$\int_0^t R_4(t-x)\, \overline{F}_2(x)\, V_1(x)\, dU_2(x) + \int_0^t R_4(t-x)\, \overline{F}_2(x)\, U_2(x)\, dV_1(x)$$

$$R_3(t) = \bar{F}_1(t)\,\bar{U}_1(t) + \int_0^t R_1(t-x)\,G_2(x)\,\bar{U}_1(x)\,dF_1(x)$$

$$+ \int_0^t R_2(t-x)\,G_2(x)\,\bar{F}_1(x)\,dU_1(x) + \int_0^t R_2(t-x)\,\bar{F}_1(x)\bar{U}_1(x)\,dG_2(x) \tag{4}$$

$$R_4(t) = \bar{F}_1(t)\,\bar{U}_1(t) + \int_0^t R_1(t-x)\,V_2(x)\,\bar{U}_1(x)\,dF_1(x) +$$

$$\int_0^t R_2(t-x)\,\bar{F}_1(x)\,V_2(x)\,dU_1(x) + \int_0^t R_2(t-x)\,\bar{F}_1(x)\bar{U}_1(x)\,dV_2(x) \tag{5}$$

Taking the Laplace - Stieltjes (Ls) transform of these equations (1-5), we get the (Ls) transform of the distribution of TFSF.

$$R(s) = -\int_0^\infty e^{-st}\,dR(t) = \frac{L_1(\alpha_{11}L_2 + \alpha_{21}L_3) + L_4(\alpha_{11}L_5 + \alpha_{21}L_6)}{L_3L_5 - L_2L_6} \tag{6}$$

Where

$L_1 = \gamma_{11} + \gamma_{21} + c_{11}L_7 + (d_{11}+e_1)L_8$; $L_2 = c_{22}c_{12} + (d_{22}+\tilde{e}_2)c_{21}$

$L3 = 1 - c_{22}(d_{12}+\tilde{e}_1) - (d_{22}+\tilde{e}_2)(d_{21}+e_2)$; $L_4 = \gamma_{11} + \gamma_{21} + c_{22}L_7 + (d_{22}+\tilde{e}_2)L_8$

$L_5 = 1 - c_{11}c_{12} - (d_{11}+e_1)c_{21}$; $L_6 = (d_{12}+\tilde{e}_1)c_{11} + (d_{11}+e_1)(d_{21}+e_2)$

$L_7 = \alpha_{11} + \alpha_{21} - c_{12} - d_{12} - \tilde{e}_1 - 1$; $L_8 = \alpha_{11} + \alpha_{21} - c_{21} - d_{21} - e_2 - 1$;

$\alpha_{11}(s) = \int_0^\infty e^{-st}\,\bar{F}_1(t)\,dU_1(t)$; $\alpha_{21}(s) = \int_0^\infty e^{-st}\,\bar{U}_1(t)\,dF_1(t)$

$\gamma_{11}(s) = \int_0^\infty e^{-st}\,\bar{F}_2(t)\,dU_2(t)$; $\gamma_{21}(s) = \int_0^\infty e^{-st}\,\bar{U}_2(t)\,dF_2(t)$

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$$C_{11}(s) = \int_0^\infty e^{-st} G_1(t)\,\overline{U}_2(t)\,dF_2(t) \quad ; \quad C_{12}(s) = \int_0^\infty e^{-st} G_2(t)\,\overline{U}_1(t)\,dF_1(t)$$

$$C_{21}(s) = \int_0^\infty e^{-st} V_2(t)\,\overline{U}_1(t)\,dF_1(t) \quad ; \quad C_{22}(s) = \int_0^\infty e^{-st} V_1(t)\,\overline{U}_2(t)\,dF_2(t)$$

$$d_{11}(s) = \int_0^\infty e^{-st} G_1(t)\,\overline{F}_2(t)\,dU_2(t) \quad ; \quad d_{12}(s) = \int_0^\infty e^{-st} G_2(t)\,\overline{F}_1(t)\,dU_1(t)$$

$$d_{21}(s) = \int_0^\infty e^{-st} \overline{F}_1(t)\,V_2(t)\,dU_1(t) \quad ; \quad d_{22}(s) = \int_0^\infty e^{-st} \overline{F}_2(t)\,V_1(t)\,dU_2(t)$$

$$e_1(s) = \int_0^\infty e^{-st} \overline{F}_2(t)\,U_2(t)\,dG_1(t) \quad ; \quad \tilde{e}_1(s) = \int_0^\infty e^{-st} \overline{F}_1(t)\,U_1(t)\,dG_2(t)$$

$$e_2(s) = \int_0^\infty e^{-st} \overline{F}_1(t)\,U_1(t)\,dV_2(t) \quad ; \quad \tilde{e}_2(s) = \int_0^\infty e^{-st} \overline{F}_2(t)\,U_2(t)\,dV_1(t)$$

The mean time T_m to the system failure (MTSF) $= \left. \dfrac{-dR(s)}{ds} \right|_{S=0}$

In case of two similar units $G_1(t) = G_2(t)$;

$$V_1(t) = V_2(t) = V(t) \; ; \; U_1(t) = U_2(t) = U(t) \; ; \; \alpha_{11} = \gamma_{11} = \gamma_1 ,$$

$$\alpha_{21} = \gamma_{21} = \gamma_2 \; ; \; C_{11} = C_{12} = C_1 \; ; \; C_{21} = C_{22} = C_2 ; \; d_{11} = d_{12} = d_1 \; ;$$

$$d_{21} = d_{22} = d_2 \; ; \; e_1 = \tilde{e}_1 = e_1 \; ; \; e_2 = \tilde{e}_2 = e_2 \; ;$$

The mean time T_m for this case has the form

$$T_m = a(0) \left[1 + \frac{\gamma_2(0)\left[(1+d_1(0) - d_2(0) + e_1(0) - e_2(0)\right] + \gamma_1(0)\left[(1-C_1(0)+C\right.}{(1-C_1(0))\,(1-d_2(0)-e_2(0)) - C_2(0)\,(d_1(0)+e_1(0))} \right.$$

Which is consistent with [7] , where $a(0) = \int_0^\infty \overline{F}(t)\,\overline{U}(t)\,dt$.

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THE LIMITING FAILURE TIME DISTRIBUTION OF THE SYSTEM

If the operation time T_0 of the main equipment is more greater than the ime of repair T_R, and the time of inspection T_I

.e. $T_0 \gg T_R$, $T_0 \gg T_I$, then there exist ν, $\varepsilon_{\nu i}$ $(i = 1,2,3,4)$

uch that

$$\bar{G}_{i\nu}(x) = 1 - G_{i\nu}(x) < \varepsilon_{\nu i} \ ; \ \bar{V}_{i\nu}(x) = 1 - V_{i\nu}(x) < \varepsilon_{\nu j} (i=1,2;\ j=3,4), x > 0$$

hus

$$\lim_{\nu \to \infty} \bar{G}_{i\nu}(x) = 0 \ ; \ \lim_{\nu \to \infty} \bar{V}_{i\nu}(x) = 0 \ (i = 1,2) ,$$

$$\int_0^\infty \bar{G}_{1\nu}(x)\, dF_2(x) \xrightarrow[\nu \to \infty]{} 0 \ ; \ \int_0^\infty \bar{G}_{2\nu}(x)\, dF_1(x) \xrightarrow[\nu \to \infty]{} 0 \ ; \ \int_0^\infty \bar{V}_{1\nu}(x)\, dF_2(x) \xrightarrow[\nu \to \infty]{} 0 \ ;$$

$$\int_0^\infty \bar{V}_{2\nu}(x)\, dF_1(x) \xrightarrow[\nu \to \infty]{} 0 \tag{9}$$

Hence, the following theorem is true .

Theorem : In the case of a constant time preventive maintenance policy where

$$U_1(t) = U_2(t) = U(t) \ ; \ U(t) = \begin{cases} 1 & t \geq T \\ 0 & t < T \end{cases} \tag{10}$$

and

$$\alpha_\nu = \Big[F_1(T) \int_0^\infty \bar{G}_{1\nu}(t)\, dF_2(t) + F_2(T) \int_0^\infty \bar{G}_{2\nu}(t)\, dF_1(t) + \bar{F}_1(T) \cdot \int_0^\infty \bar{V}_{1\nu}(t)\, dF_2(t) + \bar{F}_2(T) \int_0^\infty \bar{V}_{2\nu}(t)\, dF_1(t) \Big] \xrightarrow[\nu \to \infty]{} 0 , \tag{11}$$

the distribution function of the operating time of the system tends to an exponential one .

Proof :

From (6),(7) and (10), the Laplace transform of the distribution function of the operating time of the system given as a function of α_ν^s , takes the form

$$R(\alpha_\nu s) = \frac{I_1 I_2 + I_3\, I_4 - I_5\, I_6 - I_7\, I_8 + I_6 + I_8 + I_9 + I_{10} - I_4 - I_{12} + o(\alpha_\nu^2}{I_1\, I_2 + I_3 I_4 - I_5\, I_6 - I_7 I_8 + I_6 + I_8 + I_9 + I_{10} - I_4 - I_{11} + o(\alpha_\nu^2}$$

where

$$I_1 = 1 - \int_0^T e^{-\alpha_\nu st}\, G_{1\nu}(t)\, dF_2(t) \int_0^T e^{-\alpha_\nu st}\, G_{2\nu}(t)\, dF_1(t)$$

$$I_2 = 1 - \int_0^\infty e^{-\alpha_\nu st}\, V_{1\nu}(t)\, dF_2(t) \int_0^\infty e^{-\alpha_\nu st} V_{2\nu}(t)\, dF_1(t)$$

$$I_3 = 1 - \int_0^T e^{-\alpha_\nu st} V_{1\nu}(t)\, dF_2(t) \int_0^T e^{-\alpha_\nu st}\, V_{2\nu}(t)\, dF_1(t)$$

$$I_4 = 1 - \int_0^\infty e^{-\alpha_\nu st}\, G_{1\nu}(t)\, dF_2(t) \int_0^\infty e^{-\alpha_\nu st}\, G_{2\nu}(t)\, dF_1(t)$$

$$I_5 = 1 - \int_0^T e^{-\alpha_\nu st} V_{1\nu}(t)\, dF_2(t) \int_0^T e^{-\alpha_\nu st} G_{2\nu}(t)\, dF_1(t)$$

$$I_6 = 1 - \int_0^\infty e^{-\alpha_\nu st}\, G_{1\nu}(t)\, dF_2(t) \int_0^\infty e^{-\alpha_\nu st}\, V_{2\nu}(t)\, dF_1(t)$$

$$I_7 = 1 - \int_0^T e^{-\alpha_\nu st}\, G_{1\nu}(t)\, dF_2(t) \int_0^T e^{-\alpha_\nu st} V_{2\nu}(t)\, dF_1(t)$$

$$I_8 = 1 - \int_0^\infty e^{-\alpha_\nu st}\, V_{1\nu}(t)\, dF_2(t) \int_0^\infty e^{-\alpha_\nu st} G_{2\nu}(t)\, dF_1(t)$$

$$I_9 = \int_0^T e^{\alpha_\nu st}\, V_{1\nu}(t)\, dF_2(t) \int_0^\infty e^{-\alpha_\nu st}(V_{2\nu}(t) - G_{2\nu}(t))\, dF_1(t)$$

$$I_{10} = \int_0^\infty e^{-\alpha_\nu st}(V_{1\nu}(t) - G_{\nu}(t))\, dF_2(t) \int_0^T e^{-\alpha_\nu st} V_{2\nu}(t)\, dF_1(t)$$

$$I_{11} = \alpha_\nu s \left[\int_0^\infty e^{-\alpha_\nu st} dF_1(t) \int_T^\infty e^{-\alpha_\nu st} \bar{F}_2(t)\, dt + \int_0^\infty e^{-\alpha_\nu st} dF_2(t) \cdot \int_T^\infty e^{-\alpha_\nu st}\, \bar{F}_1(t)\, dt \right]$$

$$I_{12} = \alpha_\nu s \left[\int_0^T e^{-\alpha_\nu st} \bar{F}_2(t)\, dt + \int_0^\infty e^{-\alpha_\nu st} \bar{F}_1(t)\, dt \int_0^\infty e^{-\alpha_\nu st} dF_2(t) + \int_T^\infty e^{-\alpha_\nu st} \bar{F}_2(t) dt \, . \int_0^\infty e^{-\alpha_\nu st} dF_1(t) \right]$$

From the above assumptions (9) , we have

$$\left.\begin{aligned} &\left[I_1 I_2 + I_3 I_4 - I_5 I_6 - I_7 I_8 \right] / \alpha_\nu \xrightarrow[\alpha_\nu \to 0]{} 0 \quad , \\ &\left[I_6 + I_8 + I_9 + I_{10} - I_4 \right] / \alpha_\nu \xrightarrow[\alpha_\nu \to 0]{} 1 + s\,(T_1 + T_2) \quad , \\ &(I_{11} / \alpha_\nu) \xrightarrow[\alpha_\nu \to 0]{} s\,(\delta_1 + \delta_2) \; ; (I_{12} / \alpha_\nu) \xrightarrow[\alpha_\nu \to 0]{} s\,(T_1 + T_2) \; . \end{aligned}\right\} \qquad (13)$$

where

$$T_1 = \int_0^\infty t dF_1(t) \quad ; \quad T_2 = \int_0^\infty t dF_2(t) \quad ; \quad \delta_1 = \int_T^\infty \bar{F}_1(t)\, dt \; ; \; \delta_2 = \int_T^\infty \bar{F}_2\,(t) dt$$

It is evident from (12) and (13) that

$$R\,(\alpha_\nu s) \xrightarrow[\alpha_\nu \to 0]{} \frac{1}{1 + s\,(\lambda_1 + \lambda_2)}$$

where

$$\lambda_1 = \int_0^T \bar{F}_1(t)\, dt \quad , \quad \lambda_2 = \int_0^T \bar{F}_2(t) \quad dt$$

which completes the proof of the theorem .

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